Towards an effective poroelastic model for fracture media

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# Abstract

\* The concept of effective media as the homogeneous equivalent, keeping the 4 independent parameters.

\* Effective stress based on Biot coefficient is valid for porous media, but not for plasticity, fractures etc.

\* Fractured media micromechanics strongly differ from homogeneous porous media.

\* Fracture behavior strongly influences the equivalent macroscopic mechanical behavior

\* Fractures are difficult to characterize in lab, numeric effective models look more interesting

\* We use inhouse simulator, 3D, discrete fractures to assess macroscopic mechanical impact of fractures

\* We first present a thorough validation of the simulator and then use the simulator in automated batch runs for calculating the effective parameters

\* We can see increase of the biot coefficient, decrease in drained bulk modulus and increase in biot modulus etc

\* The validity is bound by small strains and linear elastic behavior of the whole system.

*Keywords: Naturally Fractured Reservoirs; Numerical methods; Linear poroelasticity; Biot consolidation; Skempton coefficient.*

# INTRODUCTION

## Motivation, objectives

Introduction to Naturally fractured reservoirs, importance on energy industry. Poromechanics and scale. The objective of this paper is to discuss effective media approaches and the homogenization theory for fractured poroelastic media. Mechanical assessment of naturally fractured reservoirs in the macroscale.

Concept of effective stress and its misuse (sometimes we see people talking on Biot=1 for a fracture. I think it is pointless to talk about Biot of a fracture, as it is not a porous media. I’d rather call it a pseudo-effective-stress coefficient used for scaling purpose.).

Assume that fracture properties respond to the difference between the total stress and the pressure. (Bouteca), while intact porous media volumetrics respond to an effective stress coefficient (Biot-willis) greater than the porosity, smaller than one. Important ideas from (Boutéca and Guéguen, 1999) - discuss the effective stresses in different contexts

Previous work and difficulty of large size lab testing, we do not recommend using lab results straight into field scale models. Reservoir geomechanifcs. That lab is not able to characterize fractures effectively. The fracture parameters are always going to be uncertrain. So we need can make use of a numerical approach, under uncertainties, dealing with ranges and plausible values.

We know rock mechanics is highly nonlinear, but we can use locally linear small strain models to assess the dependency among the parameters, perform sensitivity and sanity check the ranges of validity of each parameter. Assume that the fractures are elastic and that the continua are within the elastic region (no fracture propagation of plasticity)

Our goal is to recommend approach to assign effective parameters to field-scale geomechanical models, normally used to investigate reservoir mechanical response, like fault reactivation, subsidence and quasi-static stress evolution. Provide a rule-of-thumb recomendation from the sensitivity analysis for mechanical homogenization of a fractured media into an effective poroelastic media.

## Pros and cons of our approach

Advantage:The inhouse simulator we have is fully automated and integrated with large computer cluster, meaning that we are able to make thousands of runs in a few hours he isotropic poroelastic media has 4 independent parameters. We address the 4 of them: K, , M,.

One of the limitations is trying to assign isotropic parameters to a media that is not isotropic anymore, as of the fractures. We have good references for that in the mechanical engineering literatyre. We believe locally homogeneous model for a strucutre we know that is not homogeneous nor isotropic in any scale is an approximation.The locally isotropic assumption is common in simulators for simplicity and performance.

Remember that we have a spatially discretized models, so that the anisotropy and heterogeneity is dealt with in the macro scale. We are assuming possible to represent the REV as isotropic and homogeneous. Under uncertainties, the assumption is ok as long as we identify the tradeoffs and risks under uncertainties. Ex: eventhough we derive equations assuming homogeneous isotropy, the models must add anisotropy uncertainties. We provide error measurements for anisotropic approximations. We need to simulate the fractured media and the unfractured media with the upscaled parameters.

## Paper flow

(1) development and validation of inhouse finite element simulator, using DFM explicit mapping of fractures. (2) automated reproducible framework for uncertainty assessment. (3) Results of simple isotropic fracture. (4) Results of more fractures (5) Monte carlo for isotropic (6) Anisotropic – measure the error in the approximation

Table 1 – Symbols and relevant definitions for linear poroelasticity

|  |  |  |
| --- | --- | --- |
| Symbol | Description | Unit |
|  | Cauchy (total) stress tensor (positive for tension, negative for compression) |  |
|  | Biot effective stress (positive for tension, negative for compression) |  |
|  | Terzaghi effective stress (positive for tension, negative for compression) |  |
|  | Isotropic applied stress field (positive for tension, negative for compression) |  |
|  | Biot coefficient |  |
|  | Displacement tensor |  |
|  | Isotropic shear modulus |  |
|  | Drained Young modulus |  |
|  | Isotropic Poisson ratio |  |
|  | Fluid content of the (positive for fluid added) |  |
|  | Hidraulic conductivity |  |
|  | Gravity force |  |
|  | Fluid viscosity |  |
|  | Rock intrinsic permeability |  |
|  | Ratio between intrinsic permeability and fluid viscosity |  |
|  | The permeability and the ration between permeability and viscosity in the fracture domain |  |
|  | Fluid density |  |
|  | Biot modulus |  |
|  | Fluid storage coefficient in constant strain |  |
|  | Pore pressure, positive for compression. |  |
|  | Drained bulk modulus. |  |
|  | Undrained bulk modulus () |  |
|  | Skempton coefficient |  |
|  | Variation of volume of voids from configuration to configuration |  |
|  | Variation of volume of solids from configuration to configuration |  |
|  | Bulk volume of the |  |
|  | Representative elementary volume |  |
|  | Poroelastic stiffness fourth-order tensor |  |
|  | Terzaghi effective cohesive stiffness normal to the fracture surface |  |
|  | Total cohesive stiffness tangential to the fracture surface, expressed in the fracture local coordinate system |  |
|  | Total cohesive traction in the fracture surface expressed in the fracture local coordinate system |  |
|  | Terzaghi effective cohesive traction normal to the fracture surface. |  |
|  | Total cohesive traction in the fracture surface expressed in the global coordinate system |  |
|  | Biot effective cohesive traction in the fracture surface expressed in the global coordinate system |  |
|  | Terzaghi effective cohesive traction in the fracture surface expressed in the global coordinate system |  |
|  | Elevation |  |
|  | Time derivative |  |
|  | Strain tensor. |  |
|  | Volumetric strain |  |
|  | Dirak delta |  |
|  | Elevation head |  |
|  | Unitary normal and tangential vectors to the matrix faces, expressed in the global coordinate system |  |
|  | Rotation matrix from local () to global () coordinate system. |  |
|  | Jump operator applied to the displacement vector |  |
|  | Displacement vector across a discontinuity. The superscript and identify the opposing sides of the discontionuity. |  |
|  | The aperture normal to the fracture surface. |  |

# FORMULATION

This section presents the mathematical formulation for the in-house simulator in the strong form. The numerical domain is split in a *continua domain* to represent the fluid flow and mechanical deformation of the intact rock, and a *fracture domain* to represent fluid flow and mechanical equilibrium along rock joints.

The continua constitutive law follows entirely the linear poroelasticity as originally proposed by (Biot, 1941) and more recently comprehensively described in text books (Cheng, 2016; Wang, 2000). In the fracture domain, the models for mechanical equilibrium and flow continuity in the fracture domain follow principles of local linearity, as originally proposed by (Cundall, 1971; Goodman et al., 1968).

Symbols and fundamental formulations of poroelasticity used in this work are presented in Table 1 for clarity. This work uses indicial notation for vectors and tensors to favor implementation of low level computer routines. Repeated indices in a product represent the Eistein summation throughout the dimensions in use and can be promptly translated into looped iterations. When omitted, are independent iterators for the global coordinate system .

## Continua domain: linear poroelasticity and flow continuity

In linear poroelasticity, the effective stress tensor is linearly related to the strain tensor by

, (1)

where is the stiffness tensor which, for linear elastic media is

. (2)

The mechanical equilibrium condition for a is given by the divergence-free condition of the Cauchy total stress tensor, that is

. (3)

Using relations in Table 1 and observing the symmetric properties of it is clear that

, (4)

which are said to be the constitutive partial differential equations for the mechanical equilibrium of the continua.

To achieve flow continuity, the rate of increment of fluid content must balance the fluid entering minus leaving the continua , that is

. (5)

Darcy’s law states that the fluid flow through the is given by the inner product of the hydraulic conductivity tensor and the gradient of the hydraulic head:

. (6)

Considering horizontal single-phase isotropic flow, gravity is neglected, and the flow equilibrium constitutive partial differential equation becomes

(7)

## Fracture domain: flow continuity and mechanical equilibrium

To achieve flow continuity in the fracture domain, the rate of increment of the normal aperture must balance the fluid entering minus the leaving the fracture , that is

. (8)

This work assesses a fracture sample on steady state, so that the fluid flow inside the fracture is in equilibrium with the surrounding continua. In the direction normal to the fracture surface, the fluid flow is negligible.

For that, a constant tangential permeability is assumed in the fracture domain, and the fluid flow becomes

(9)

In the direction normal to the fracture surface, the fluid is free to flow inside the fracture, and no pressure gradient is expected. However, the normal flow is an important boundary condition to couple with the continua and must still be present in the following:

(10)

The mechanical equilibrium in the fracture domain is controlled by cohesive forces described in the local coordinate system . The parameters are the cohesive stiffness in the normal and tangential directions to control the fracture mechanics. For the present work, in all cases. Hence, the fracture Terzaghi effective cohesive traction in the local coordinate system is

(11)

where

  . (12)

Note that a zero stiffness represent a cohesion free fracture, meaning that the fracture surfaces do not interact. However, the fluid pressure in side the fracture still acts as a total normal stress with or, equivalently, .

The rotation tensor must be applied to rotate (11) into the global coordinate system. As the aperture in the local coordinate system is

, (13)

the Terzaghi effective cohesive traction in the global coordinate system is

, (14)

and observing of the orthogonal nature of the rotation matrix,

, (15)

the traction acting on the fracture surface in global coordinates is

. (16)

Lastly, the Biot effective cohesive traction on the face of the fracture as

. (17)

The final expression for the is

(18)

# NUMERICAL SOLUTION

This section describes the numerical strategy to approximate the partial differential equations described above.

The space discretization follows the Finite Element Method as in (Hughes, 2000). Numerical stability is ensured by LBB stability condition using second order basis functions for displacement variables and first order shape functions for pressure (Murad and Loula, 1994). The fracture model use elements of reduced dimension, that is, the dimension normal to the fracture surface is analytically reduced as flow and traction boundary conditions for the continua and only the tangential flow is solved numerically as 2D surface elements.

Time marching follows second-order implicit Crank-Nicholson timestepping, with proven gain in performance and accuracy compared to first-order implicit schemes during validation tests.

## Space discretization using the Finite Element Method

This section presents the formal statement of the problem in a Finite Element environment. Table 2 summarizes definitions for the development that follows. Further strict mathematical background are found in (Hughes, 2000).

Equations (4, 7,10,18) comprise the strong form of the problem, rewritten below for clarity:

on

on

on

on (19)

The derivation of the weak form for the continua domain is straightforward. The coupling to the fracture domain requires a little attention. For that, it is convenient to first write the weak form of equation (10):

Ignoring the flow normal to the fracture surface, the fluid exchange with the continua is

that is the coupling term.

The weak form that derives from the continua domain – equations (4, 7) – is

Find and , such that and

where

. (20)

Replacing and at the fracture boundary , we couple the continua and fracture domain in a single weak form:

Find and , such that and

(21)

and the Galerkin formulation is

Find and , such that and

(22)

Following the FEM workflow, the system is now split in elements and nodes to be evaluated in a linear system of equations. The iterators are introduced to represent the mesh nodes. The nodal values and their time derivatives, a linear equation system in the form of

(23)

is obtained by approximating them by a linear combination of the element shape functions as

and . (23)

The following expression describes the linear equation system , in which represents the approximation of variable at node and identifies the equation associated with the node . The systems parameters are assumed locally constant and leave the integrals.

The resulting form of the linear equation system is described below. The terms are split according to the associated physics for clarity:

Continua mechanics:

Continua flow and mass balance:

Fracture mechanics:

Fracture flow and mass balance:

Dirichlet boundary conditions:

(23)

## Time discretization using Crank-Nicholson

The time derivatives in equation (23) is linearly expanded as

where the terms is known from the previous iteration or from initial conditions, and the terms are the unknowns.

For the terms , the second-oder implicit Crank-Nicholson approximation imply that

Note that the coefficient of the unknowns comprise the tangent matrix, while the coefficients of the can be fully evaluated and sums into the right hand side vector .

## Shape functions

## Linear solver

Table 2 – Symbols and relevant definitions for the Finite Element Method framework

|  |  |
| --- | --- |
| Symbol | Description |
|  | Collection of the trial solutions for displacement. |
|  | Collection of the trial solutions for pressure. |
|  | Collection of the test solutions for displacement. |
|  | Collection of the test solutions for pressure. |
|  | Linear combination of shape functions to approximate . |
|  | Linear combination of shape functions to approximate . |
|  | Heterogeneous (non-zero) Dirichlet (natural) constraints for . |
|  | Heterogeneous (non-zero) Dirichlet (natural) constraints for . |
|  | Approximation of the jump operator across a discontinuity. The positive () superscript represents the displacement on the positive face of the fracture and the negative () represent the displacement in the negative face of the fracture. |
|  | Number of nodes in the mesh. |
|  | inner product of |
|  | norm of |
|  | The Hilbert space is defined as the collection of square integrable functions (finite norm). |
|  | inner product of |
|  | norm of |
|  | The Sobolev space is defined as the collection of functions with finite norm. |
|  | Continua domain |
|  | Fracture domain |
|  | Boundary of the continua, excluding the fracture domain |
|  | Volume of fluid flowing normal to a surface |

Second order in time shows advantage, larger timest

### Historical development in numerical models for fractured rocks

Zero-width cohesive elements

(Goodman et al., 1968) and (Cundall, 1971) seem to be the first to propose zero-thickness models for the fractures.

(Ghaboussi et al., 1973) showed a finite element implementation:

A black text on a white background

Description automatically generated

### Recent numerical models for fractured rocks, homogenization

(Poli et al., 2021) - the original simulator, now expanded to 3D. The fracture propagation feature is not in use.

(Marinelli et al., 2016) (Frey et al., 2013) - shows the implementation of the cohesive elements applied to homogenization. Instead of Skempton, they use Sε .

## Computational framework

### Mesh generation

(Geuzaine and Remacle, 2009) - the gmsh reference. Gmsh, python API

The fracture geometry and the mesh are variables of the system and must be generated automatically in every run

(Kirk et al., 2006) - the libmesh reference

(Balay et al., 2024) – PETSc web page

### Mesh generation – manipulation of the fracture elements

Split fracture elements DoF

### AUTOMATED REPRODUCIBLE FRAMEWORK

Uncertainty framework must be highly parallelized

Work in a light terminal and run in a HPC

Import results back

Python, Makefile

# ESTIMATION OF EFFECTIVE PARAMETERS

Fracture models are modeled widely for flow purposes, not so often for mechanical coupling in poroelastic aplications. Coupling: fully, one-way etc. Fracture stiffness likely locally lower than continua stiffness. It is to expect reduced effective Bulk modulus to represent the macroscale compliance.

(Min and Jing, 2003) – numerical determination of the equivalent elastic compliance tensor for fractured rock masses. Proposed the BMM, using Itasca UDEC to simulate

(Grechka and Kachanov, 2006)– effective elasticity of fractured rocks, focus on geophysics. The larger fractures dominate the mechanical effect because the effect is proportional to the radius to the cubic

(Chen et al., 2020) – effective stress coefficient of saturated fractured rocks. They use two methods: Equivalent strain method (ESM) and Bulk modulus method (BMM). Chen et al. (2020) proposed the ESM, the same that (De Simone et al., 2023) uses.

(De Simone et al., 2023) - homogenization of biot and skempton

Describe our approach based on the parameters definitions.

Describe the parameters , , ,

Include definition as background to the mass balance validation

# RESULTS

## MODEL VALIDATION

### Terzaghi consolidation

(Von Terzaghi, 1923)

A graph of different colored lines

Description automatically generated

### Mandel’s problem

(Mandel, 1953)

A graph of different colored lines

Description automatically generated

### Sneddon solution

(Sneddon, 1946)

A graph of a rainbow

Description automatically generated

A blue and orange gradient

Description automatically generated

### Mass balance of an open fracture

A blue and red background with a curved object

Description automatically generated with medium confidence

## ISOTROPIC FRACTURE

## A wireframe of a cube Description automatically generatedA colorful triangular object with many triangles Description automatically generated with medium confidence

## 

## ANISOTROPIC FRACTURE

*(Not yet run)*

## MONTE CARLO

## A group of blue and green graphs Description automatically generated

## DIFFERENCE IN RESULTS USING LABORATORY OR EFFECTIVE

### Subsidence

### Stresses around well

# DISCUSSION AND FINAL REMARKS

## Laboratory results must be upscaled in a NFR into an effective poroelastic model

## There are clear trends for each parameter of the effective model

## Fracture orientation does not matter for Biot and Skempton, intensity does

## Propose ranges for parameters within the studied models

## Fracture parameters must be addressed as uncertainties of the system

## The lower bound of the ranges should be higher than the laboratory results

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